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Perspective and the Scope of Optics

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1. Introduction

In some senses optics and perspective are very different: optics deals with vision: linear perspective involves representation. A person may be excellent at seeing spatially, and still be hopeless when it comes to drawing spatially in perspective. Yet etymologically, the Latin term for optics and linear perspective is the same (*perspectiva*). Historically the discovery of linear perspective in the Renaissance brought no clear distinctions between theories of seeing and methods of representation.[1] The terms for optics and perspective remained effectively interchangeable. One important reason for this was that the new perspective instruments which served as drawing aids were also tools for the verification of sight, which became linked with new criteria for proper vision and changed optics from a study of the eye to include extensions of sight. Hence the discovery of perspective in the fifteenth century, the upsurge in measuring instruments during the sixteenth century and the rise of telescopes and microscopes in the seventeenth century are three interconnected developments. Perspective instruments did not simply affect painting practice: they extended the scope of optics, changed the criteria for veridical vision, transformed the very process of objectivity, and hence affected western science as much as art.

2. Ancient Criteria

In classical Greece, optics was primarily concerned with the eye and vision: focussing on psychological questions of how the eye is deceived. Optics was studied in at least three different disciplines.[2] One was medicine and is best known to us through the work of Galen: it concerned the anatomy of the eye and study of the usefulness of its various parts. A second was geometrical optics as found in Euclid's *Optics*. Its relation to geometry was outlined by Aristotle in the *Physics*:” While geometry studies physical lines but not qua physical, optics investigates mathematical lines but qua physical not qua mathematical.”[3]

Euclid's *Optics* dealt mainly with appearances, with what would today be termed psychological optics, i.e situations in which the eye can be tricked or deceived. It also contained four surveying propositions concerned with actual measurements of objects.[4] Related to geometrical optics was a third major discipline as Aristotle explained in the *Posterior Analytics*: " Knowledge of the fact is within the province of the natural philosopher, knowledge of the reasoned fact within that of the optician, either qua optician or qua mathematical optician."[5]

If we study what Aristotle actually does, a more complex picture emerges, namely, that natural philosophy tended to divide into two disciplines: physics as in the *Meteorology* and philosophy, as in *On the Soul* (*De anima*) which dealt with theories of how the eye sees. This philosophical context accounts for why theories of vision had played such an important part in Plato's *Timaeus* and continue to do so in Lucretius' *On the Nature of Things*. Lucretius is of particular interest because he made a list of optical illusions. These were typically illusions of the unaided eye: how, for instance, the square towers of a city appear rounded at a distance[6] or how a portico:

> Albeit it stands well propped from end to end
> On equal columns, parallel and big
> Contracts by stages in a narrow cone,
> When from one end the long, long whole is seen,
> Until conjoining ceiling with the floor,
> And the whole right side with the left, it draws
> Together to a cone's nigh-viewless point.[7]

Lucretius mentioned only the perception of this phenomenon, not its representation. He was not concerned with linear perspective: only with psychology of vision and if such cases left him unwilling to concede that eyes were cheated, his conclusions were not optimistic: "'Tis after all the reasoning of the mind that must decide; nor can our eyeballs know The nature of reality".[8]

It is noteworthy that the same Plato, who so complained of optical illusions in the *Sophist* and the *Republic*, appears to have inspired in his successor, Carneades,[9]distinctions between different categories of vision as part of a quest for criteria to get beyond these visual illusions; a goal taken up by the Stoic school, by Ptolemy in his *Optics* and subsequently by Ibn al-Haytham in his great *Book of Optics*.

The late classical period also played its role in expanding the scope of visual science. Hero of Alexandria, for instance, divided vision into: 1) optics, the study of vision proper;
2) dioptrics, the study of dioptras and sighting instruments and 3) catoptrics or the study of mirrors.[10] This provided a further context for linking study of the unaided eye with the use of instruments, particularly surveying instruments.

3. Mediaeval Developments

The Arabic tradition responded in different ways to this Greek heritage. In the case of Ibn al-Haytham (Alhazen), for instance, it is known that the writings of Ptolemy formed a point of departure for his own original studies.[11] Whereas ancients such as Plato or Lucretius had despaired of being able to trust the eye, Ibn-al Haytham was optimistic that one could, with careful study, reach some truthful conclusions.[12] He established that vision occurs through images coming to the eye (intromission), and made a much more detailed study of conditions under which the eye is and is not deceived.[13] Fundamental in this context was his reliance on visual evidence based on observation which, as Sabra has acutely noted[14] derived from the astronomical tradition and led Ibn al-Haytham to develop an experimental approach to visual truth. Ibn al-Haytham was conscious of the philosophical importance of veridical vision and recognized the role of psychological factors in this process. Even so his experiments were mainly limited to determining conditions under which the unaided eye sees accurately.

Meanwhile, another strand of the Arabic tradition took up the links between optics and surveying implicit in Euclid and explicitly mentioned by Hero of Alexandria. The tenth century philosopher, Al-Farabi, described optics in terms of surveying problems such as heights of trees and mountains, widths and depths of valleys and in this context also referred to instruments:

> In general every visible magnitude of which the size or distance from something else we seek to know [can be determined] sometimes by means of instruments which are made for guiding the passage of sight so that it may not err and sometimes without such instruments.[15]

In the Latin West the ideas of Al-Farabi were spread by Gundissalinus and became part of a new approach to optics. Witelo, writing in the 1270's, specificallly described instruments such as the astrolabe and quadrant as being for "the certification of sight",[16] i.e. instruments were now used to check and measure optical evidence. This helps to account for increasing links between optics and surveying in the fourteenth century[17], and indeed it can hardly be a co-incidence that these same links are evident among the authors on perspective in the fifteenth century, notably Francesco di Giorgio Martini and Leonardo da Vinci.

Related to the above was another shift which had been underway since Antiquity. In his *Optics*, Ptolemy had dealt with direct, reflected and refracted vision. Ibn al-Haytham and Witelo continued this tradition. Indeed their works contained early examples of instruments which cause objects to appear closer or further. Roger Bacon took this approach further, claiming that one could arrange refraction and reflection of images in any way one wants such that:

> we see an object close or far and hence from an incredible distance we read very minute letters and we can count [particles of] dust and sand because of the angle under which we see them, for distance only plays an accidental role in vision of

> such objects, but the angle of vision [plays a fundamental role]. And hence a boy can appear gigantic and a man can appear [the size of] a mountain.[18]

Whether or not Bacon was able to construct such instruments in the latter thirteenth century remains a matter of debate. What interests us, however, is that his conception of optics (*perspectiva*) is not limited to the unaided eye. He assumes that it includes instruments; that optics implicitly includes extensions of the eye. The discovery of perspective along with developments in surveying and astronomy made these trends explicit.

4. Renaissance Perspective

Panofsky, as a neo-Kantian, insisted that the discovery of linear perspective in the fifteenth century was primarily a breakthrough in theory.[19] However practice played an equally fundamental role. Brunelleschi's perspective demonstrations would have been impossible without instruments.[20] The humanist, Leon Battista Alberti, who gave the first extant description of perspective in his *On Painting*, mentioned theoretical aspects, but emphasized the practical values of the veil (*velo*) for vision:

> Nothing can be found, so I think, which is more useful than that veil which among my friends I call an intersection.... This veil can be of great use to you. Firstly it always presents you the same unchanged plane. Where you have placed certain limits, you quickly find the true cuspid of the pyramid. This would certainly be difficult without the intersection....The veil will be as I said very useful to you since it is always the same thing in the process of seeing.[21]

Alberti insists that this instrument is equally important for representation: ”I do not believe that infinite pains should be demanded of the painter, but paintings which appear in good relief and a good likeness of the subject should be expected. This I do not believe can ever be done without the use of the veil.”[22]

Hence for Alberti the veil which functions as a perspectival window is much more than a handy tool. It is essential both for proper vision and representation. The idea that instruments are necessary for drawing is pursued in Alberti's *Description of the City of Rome* where he notes that such instruments are now the fashion and that he himself has been engaged in making such instruments.[23] Filarete, the next author to write on perspective, cites Alberti and is even more insistent on the role of instruments:

> From these bodies, as I have said, various measures are born, since they have various forms. And all as I have said are composed of surfaces and lines and points and are known by the separations of these lines and points as I have said.
> And from such bodies it has been found that there are instruments with which, even if nature herself made them, one is able to reduce them systematically and make them properly, instruments which, as I have said, if one wishes to work with them, one cannot err, because they are proper.

> Hence whoever wishes to make a square body their task is to have one of these instruments or if you wish measures, and this one is called a carpenter's rule, without which one cannot make a square....
> In wanting to make a sphere one cannot do it properly without a compass or a circle. And this is the other instrument without which one cannot do.[24]

Here again there is the claim that instruments are an essential part of drawing properly. Filarete digresses briefly to note a rule of thumb method without the use of instruments, but then considers a case where one needs these instruments because the eye cannot judge properly without them:

> Now in wishing to construct this square and circle in foreshortening, that is by the demonstration of drawing, where the parts, even though they be equal, cannot appear so to the eye because it cannot judge them all, and yet they are [equal], such that in order to do this it is a question of taking the position of this single point which you will take to be the eye and the visual ray [and] from which one will subsequently draw lines.[25]

In the next chapter, Filarete describes the use of the same instruments (carpenter's rule and compass) in making drawings of buildings. He then describes the use of the perspectival window or veil for this purpose, noting once again that what one draws on the window will be different from that which the eye expects.[26]

Piero della Francesca's *On the Perspective of Painting* (c.1480) is the most famous mathematical treatise on the subject in the fifteenth century. Piero begins by defining perspective as having five parts: vision, that is the eye; the form of the object seen; distance; the lines going from the object to the eye, and the intersection between the eye and the object seen at which one wants to draw things. Each of these he explains in more detail including the intersection: “in which the eye describes things proportionately with its rays and is able in this to judge their measure. And if there were not this intersection one would not know how much objects were diminished, such that they could not be demonstrated”.[27]

Piero returns to these ideas in the introductory section to book three where he notes that many persons criticize perspective because they do not understand the force of its lines and that he therefore feels obliged to show the extent to which this science is necessary for painting:

> I say that that the word perspective sounds as if to say objects seen from afar, represented under certain given intersections with proportions according to the quantities of their distances, [and] without which one cannot draw anything properly. And since painting is nothing else than the demonstration of surfaces and bodies decreased or increased at the intersection, positioned in accordance with the way in which true objects seen by the eye present themselves on the said intersection, and yet of every quantity one part is always closer to the eye than the other, and the closer presents itself at a greater angle than the more remote one at

> the assigned intersection; and since the intellect is unable of itself to judge their measure, that is by how much the one is closer and by how much the one is further, therefore I say that perspective is necesssary, which discerns all quantities proportionately as a true science, demonstrating the diminution and the augmentation of every quantity by the force of lines.[28]

The Greeks had known that the eye was deceived by illusions and had used this to argue against its reliability as a sense. Romans such as Lucretius had resisted this conclusion but found no alternative. Ptolemy and Ibn al-Haytham, usually assuming situations involving an unaided eye, had sought better criteria for determining when vision could or could not be certified. Piero's position is quite different. He is convinced that the unaided eye will always be deceived and argues that perspective, as a method which uses physical intersections, is as essential for verifying proper vision as it for representation. Instruments such as surveying rods and perspectival windows may be drawing aids for Piero: they are also a fundamental tool in the certification of sight. Hence if Piero introduces adjectives to distinguish between the perspective of painting and optics, he uses instruments in such a way that perspective and optics remain interdependent and inseparable. Leonardo da Vinci takes these ideas even further in the *Manuscript A* (c.1492) when he claims that: “Perspective is nothing else than seeing a site behind a flat transparent pane on the surface of which are marked all the things which are behind that glass and which can be conducted by means of pyramids to the point of the eye and these pyramids intersect the said pane.” [29]

Elsewhere in the same manuscript he claims that: “Perspective is a demonstrative means whereby experience confirms that all things send their similitudes to the eye by pyramidal lines.”[30] Hence, for Leonardo optics, perspective and instrumental demonstration are intimately linked. Leonardo's contemporary, Luca Pacioli, expresses analogous ideas in his *Summa* (1494). He associates perspective with proportion, claiming that painters such as Piero della Franceesca, Bellini, Botticelli and Melozzo da Forli needed these and instruments such as rulers and compasses in order to achieve the divinity of their works. Moreover: “With the mechanical arts, considering all the exercises and trades, one does not see faithfully by the eye alone. If you take from their hands the square and compass with their proportion, they do not know what they are doing.”[31]

Caporali in his edition of Vitruvius (1536) also emphasizes this link between instruments, measurement and perspective. He notes, for instance that the compass: “is more necessary for the measurements of geometry and perspective than any other instrument because with this all lineal demonstrations are measured and the angular things to which one extends the termination of lines and divisions as is well known to the expert line makers who are especially perspectivists.”[32]

In Caporali's description, perspectivists are mathematicians concerned with quantitative demonstrations. It is no wonder then that scientific individuals concerned with mathematics, such as Commandino, Benedetti, Guidobaldo del Monte, Stevin, Desargues and Pascal became involved with perspective in the century that followed. Nor is it a coincidence that some of the same individuals who wrote treatises on perspective were also connected with the development of the proportional compass. The quest for systematic representation went hand in hand with new criteria for optical veracity and a new mastery of universal measurement.

5. Astronomy, Geography and Surveying

It is important, of course, to recall that the use of instruments for the verification of observations went back to Antiquity in the fields of astronomy, geography and surveying; that this had developed in the Arabic tradition and was taken up with new intensity by Regiomontanus (Johannes Müller) precisely in the generations that perspective was evolving. For instance there is the story that Regiomonatanus decided to make Nürnberg his home base specifically because it was the best place in Europe for scientific instruments. In his treatise on the chief astronomical instruments he noted explicitly that:

> Nothing can be found that is more useful in the entire doctrine of the motions of the celestial bodies than these five astronomical instruments: the torquetum, the armillary astrolabe, the great rule of Ptolemy, the astronomical staff and the geometrical quadrant, the composition and use of which are found in these booklets since the motions of the sun, both the fixed and the erratic stars and finally the comets have been and are observed through these organs.[33]

It is noteworthy that when Regiomontanus produced a broadsheet of books which he planned to publish (c.1475) he also mentioned instruments for surveying, weights and balances, for transport of water, burning mirrors, various astronomical instruments for celestial observations and other ones for everyday use.[34]

The tradition of Regiomontanus was continued by his disciple Walther; by Pirckheimer and Schöner in the next generation; by Apian and Hartmann in the next. By the 1530's we find an ever more systematic approach to instruments. In Ingolstadt, Peter Apian, who edited the first printed edition of Witelo's classic work on optics, published one of Regiomontanus' astronomical instruments as an appendix to his *Geographical Introduction* (1532), while his *Instrument Book* (1533) contained instruments designed for all three fields: astronomy, geography and surveying. Similarly, Hartmann, who edited Peckham's *Optics*, was also responsible for a series of instruments.

Meanwhile, in Nürnberg, there was a tradition which linked perspective explicitly with measurement whence Dürer's inclusion of perspective and mathematical instruments in his *Instruction of Measurement* (1525), a tradition which continued with Rodler (1531), Hirschvogel (1543), Lautensack (1564) and Jamnitzer (1569, 1571), who produced a series of instruments and published on both perspective and surveying. His colleague, Hans Lencker, developed both perspectival and universal measuring instruments.[35] In the second half of the sixteenth century this led to new collections of scientific instruments for optics, astronomy, surveying and geography notably at the courts in Kassell, Dresden, Florence and later Prague.

Most of the instrument makers and mathematical practitioners took it for granted that these instruments were necessary in order to certify observations and optical evidence generally. Sometimes, as in Focard's edition of Bassentin's *Amplification of the Use of the Astrolabe* this intent was explicit:

> And since it is not at all possible that the sense [of sight] and the intellect can know the true quantity of the acute and variable angle, thus it would be very difficult to understand naturally the true quantity of a thing by the science of optics (*perspective*) alone. For this reason the ancient geometers and measurers invented certain artificial instruments and by means of these gave to knowing easily the quantities of things and the certitude thereof.[36]

6. Optics as Extensions of the Eye

During the Middle Ages and the Renaissance there was considerable attention to *camera obscuras*[37] in the context of analogies with the eye. Instruments to extend the capacities of the eye were much slower in evolving. One explanation has been that because lenses had the same name as lentils there was an ingrained bias against the reliability of anything other than that seen by the unaided eye and that this only changed in 1610 when Galileo had sighted the moons of Jupiter.

Our own account invites a very different reading of the evidence. During the fifteeenth century the emergence of linear perspective introduced new instrumental demonstrations of visual cones and pyramids which effectively destroyed all claims about the unaided eye's reliability. Optics, which had once been a study of eyes in isolation, now included instruments which extended the scope of vision. Moreover, as the concerns of astronomy acquired an ever more central role in the latter fifteenth century, attention shifted from the problems of direct vision to those of reflection and refraction introduced by celestial observations.

It was no coincidence therefore that Leonardo da Vinci's optical writings were intended specifically as an introduction to a great treatise on astronomy and cosmology,[38] or that Kepler, a century later, should compose his greatest optical writing on the astronomical part of optics. In short the so called revolution in optics that occured with Galileo's telescopic observations in 1610, involved something much more than suddenly over-coming scruples about the reliablity of existing lenses or acquiring better lenses. It arose also from a century old confidence that perspectival instruments were crucial in helping an otherwise inevitably deceived unaided eye. If perspective had made painting increase-ingly a discipline of representational aids; perspective made optics a science of visual aids.

Once telescopes fell within the scope of optics, microscopes soon followed: the same problem of using instruments to enlarge the size of objects, except that they are small and close rather than large and far. It is perhaps significant in this context to note that Hooke, of microscope fame, also published perspectival instruments in the P*hilosophical Transactions of the Royal Society*; that Brander wrote on perspective in the context of both telescopes and microscopes as well as surveying instruments; themes which were pursued by Lambert.

7. Conclusions

What emerges from this is that the chief significance of perspective which evolved in the fifteenth and sixteenth centuries probably lay neither in painting, where its strict rules were usually ignored; nor at first in geometry where its spatial implications were hidden by the two dimensional conventions of that tradition; but rather in focussing attention on the importance of instruments for vision as well as representation. Hence in introducing new standards for accurate drawing it also brought new criteria for optical truth. Seeing no longer involved the naked eye alone. The study of sight was now inseparable from its extensions: not just in terms of windows and rods, but also in terms of eyeglasses and combinations thereof, which led to both telescopes (or perspective glasses as they were called) and microscopes. Hence there was a good reason why Renaissance authors made no sharp distinctions between optics and perspective. The differences between vision and representation were outweighed by the perspectival instruments which remained common to both and on which both disciplines depended for their legitimation.

These connections between optics, perspective and instruments help explain why there is usually a section on optics in perspective treatises; why a perspectival window is found among the instruments at the French Academy of Sciences and why it should recur along with related perspectival instruments on the title page of a Lausanne edition of Newton's *Optics* (1740). A list of these instances could easily become an article in itself, because we are only slowly becoming aware of the extent to which the history of perspective is as central to the history of science as it is history of art.

It may be no coincidence that we are becoming aware of the significance of perspective at a time when computers are being used to reproduce perspective and even to elucidate principles of Renaissance perspective. For the extension of criteria for optical veridity from the unaided eye to include both instruments of vision and representation marked an important step in relating individual experience to a framework that is common to many and in this sense more objective. Computers essentially take this process of objectify-cation one step further and thus mark a further implication of the principles of linear perspective. Perhaps that it why perspective is witnessing a renaissance of interest. Computers are helping us to see more clearly the monumentality of the shift that perspective brought to the Renaissance: not a new way of seeing but a new instrument for recording and verifying our many ways of seeing.

Notes

[1] It is true that some Renaissance authors distinguished between the these by referring to optics simply as *perspectiva* while introducing adjectives to describe linear perspective. Hence Piero della Francesca referred to *perspectiva pingendi*; Jean Pélerin to *perspectiva artificiali*, which became *perspectiva positiva* in some editions; while Daniele Barbaro referred to *prospettiva pratica*. In German linear perspective became linked with the term

for measurement (*Messung*), hence its inclusion in Dürer's famous *Instruction in Measurement* (*Underweysung der Messung*,1525); whereas in the Netherlands it was linked with scenography (*scaenographia*). Even the other Latin term for optics (*optica*) based on the Greek (*optikn*), was frequently used to refer to linear perspective as well.

[2] For a good introduction see David C. Lindberg, *Theories of Vision from Al-Kindi to Kepler*, Chicago: University of Chicago Press, 1976.

[3] Aristotle, *Physica*, Oxford: Clarendon Press, 1930, 194a 9-12. (The Works of Aristotle, Ed. David Ross, vol. III).

[4] Cf. Wilfred R. Theisen, *The Mediaeval Tradition of Euclid's Optics*, PhD., University of Wisconsin 1971.

[5] Aristotle, *Analytica Posteriori*, Oxford: Clarendon Press, 1929, 79a 10-13 (The Works of Aristotle, Ed. David Ross, vol. I).

[6] Lucretius, *Of the Nature of Things*, trans. William Ellery Leonard, New York: E. P. Dutton, 1957, p.148.

[7] Ibid., p.151.

[8] Ibid., p. 149.

[9] Cf. Sextus Empiricus, *Against the Logicians*, trans. R. Bury, London: Heinemann, 1935, vol. 2, 1935, pp.98-99.

[10] Hero of Alexandria, *Definitiones*, ed. Heiberg, Leipzig: Teubner, 1902, pp.102-110.

[11] See: A. I. Sabra, "Ibn al-Haytham's Criticisms of Ptolemy's Optics", *Journal for the History of Philosophy*, 4, 1966, pp.145-149.

[12] See: Ibn al-Haytham, *The Optics of Ibn al-Haytham*, trans. A. I. Sabra, London: Warburg Institute, 1989, vol. 1, pp. 3-6.

[13] Ibid., pp. 6-55, particularly 51-55.

[14] A. I. Sabra, "The astronomical Origin of Ibn-al Haytham's Concept of Experiment", *Actes du Congrès International d'Histoire des Sciences*, Paris, 1968, T.IIIA (Paris 1971), pp. 133-136. This idea is developed in Sabra's exemplary edition, as in note 11, vol. 2, pp. 14-19.

For a list of Arabic manuscripts see ibid, p. lxxxiii. For a list of Latin manuscripts see the reprint of Alhazen, *Opticae Thesaurus*, ed. David Lindberg, New York: Johnson Reprint, 1972, p. xxvi. The earliest of these is now in Edinburgh (1269). Eighteen other Latin manuscripts are listed as is one Italian translation (1341), which became one of the sources for Lorenzo Ghiberti's optical writings in the early fifteenth century.

[15] Al-Farabi, *Catalogo de las ciencias*, ed. Angel Gonzalez Palencia, Madrid: , p. 37.Cf. Sabra as in note 11, vol. 2, p.lvii.

[16] Witelo, *Vitellonis Thuringopoloni opticae libri decem*, in: *Opticae Thesaurus*, Basel:Episcopios, 1572, p.217 (Liber Quintus, prop. 57): "quod totum potest fieri per astrolabium sive quadrantem vel aliud instrumentum certificationis visuum".

[17] See, for instance, Gino Arrighi, "Un estratto del De visu di M^{o} Grazia de Castellani (dal Codice Ottoboniano 3307 della Biblioteca Apostolica Vaticana", *Atti della fondazione Giorgio Ronchi*, Florence, anno XXII, gennaio-febbraio 1967, pp. 44-58.

[18] Roger Bacon, *Perspectiva*, Frankfurt: Typis Wolfgangi Richteri, sumptbus Antonii Hummii, 1616, p. 167:

> De visione refracta maiora sunt: nam de facili patet per canones supradictos, quod maxima possunt apparere minima, et e contra, et longa distantia videbuntur propinquissime et e converso. Nam possumus sic figurare perspicua et taliter ea ordinare respectu nostri visus et rerum quod frangentur radii et reflectentur quorsumcunque voluerimus et sub quocunque angulo voluerimus, videbimus rem prope vel longe et sic ex incredibili distantia legeremus literas minutissimas et pulveres ac arenas numeraremus propter magnitudinem anguli sub quo videremus, nam distantia non facit ad huiusmodi visionis nisi per accidens, sed quantitas anguli. Et sic posset puer apparere gigas et unus homo videri mons et in quacunque quantitate, secundum quem possumus videre sub angulo tanto sicut montem et prope ut volumus et sic parvuus exercitus videretur maximus et longe positus appareret prope e contra....

Roger Bacon introduced a much more complex understanding of geometry, noting, for instance, that geometry could be used to create corporeal figures in making visible biblical passages in the form of narrative paintings. By implication optical truth involved both vision and representation. See: Roger Bacon, *Opus Maius*, London, 1897, Vol.1, p.210ff:

> O quam ineffabilis luceret pulchritudo sapientiae divinae et abundaret utilitas infinita, sic haec geometricalia, quae continentur in scriptura, figurationibus corporalibus ante nostros oculos ponerentur.

For another discussion of this passage see: Klaus Bergdolt, "Bacon und Giotto. Zum Einfluss der franziskanischen Naturphilosophie auf die bildende Kunst am Ende des 13. Jahrhunderts", *Medizinhistorisches Journal*, Stuttgart, Bd. 24, Heft 1-2, 1989, pp. 25-41.

[19] Erwin Panofsky, "Die Perspektive als symbolische Form", *Vorträge der Bibliothek Warburg 1924-1925*, Leipzig, 1927, pp. 258-330.

[20] Over one hundred articles have been written on the problem of Brunelleschi and perspective in the past century. For two recent assessments see: Renzo Beltrame, "Gli esperimenti prospettici del Brunelleschi,"*Accademia nazionale dei Lincei. Rendiconti della classe di scienze morali, storiche e filologiche*, Rome, series B, vol. 28, fasc.3, March, 1973, pp. 417-468 and Martin Kemp," Science, non science and nonsense: the interpretation of Brunelleschi's perspective", *Art history*, London, vol.1, 1978, pp.134-161.

[21] Leon Battista Alberti, *De pictura*, ed. Cecil Grayson, Rome, Bari: Laterza, 1975, p.54:

> Qui adunque si dia principale opera, a quale, se bene vorremo tenerla, nulla si puo trovare, quanto io estimo, piu acommodata cosa altra che quel velo, quale io tra i miei amici soglio apellare intersegazione. ...Porgeti questo velo certo non picciola commodita: primo che sempre ti ripresenta medesima non mossa superficie, dove tu, posti certi termini, subito ritruovi la vera cuspide della pirramide, qual cosa certo senza intercisione sarebbe difficile....Adunque il velo ti dara, quanto dissi, non poco utilita ove sempre a vederla sara una medesima cosa.

The intersection is also basic to Brunelleschi's demonstration. It is worth noting that the concept of such an intersection is clearly developed by Ibn al-Haytham in the first book of his *Optics*, as in note 11, vol. 1, pp. 8-10, which was not included in the known translations into Latin or Italian, but may have been known indirectly

[22] Ibid., p.56:

> Non credo io dal pittore si richiegga infinita fatica, ma bene s'saspetti pittura quale molto molto paia rilevata e simigliata a chi ella si ritrae; qual cosa non intendo io sanza aiuto del velo alcuno mai possa.

[23] Leon Battista Alberti, *Descriptio urbis Romae*, ed. Mancini, Rome: 1890, p. 36:

> Murorum urbis Romae, et fluminis, et viarum ductus, et lineamenta, atque etiam templorum, publicorumque operum et portarum et tropaeorum situs, collocationemque ac montium finitiones atque etiam aream, quae tecto ad habitandum operta sit, uti esse per nostra haec tempora cognovimus, ex mathematicis instrumentis quam diligentissime adnotavi: eaque excogitavi, quo pacto quivis vel mediocri ingenio praeditus, bellissime et commodissime pingere, quantacumque voluerit in superficie, possit.

[24] Antonio Averlino (il Filarete), *Trattato di architettura*, ed. Anna Maria Finoli e Liliana Grassi, Milan: Edizioni il Polifilo, 1972, vol.1, p.640, (Tav.129-130, f.174r):

> Da questi corpi, come t'ho detto, nascono varie misure, come che hanno varie forme. E tutti, come ho detto, si fanno da superfice, e da linee, e da punti, e conosconsi da dispartimenti di queste linee e punti, come t'ho detto.
>
> E di questi cotali corpi s'è trovati esserci strumenti, con li quali questi corpi, benché la natura da se medesima gli abbi fatti, a volergli poi ridurre a uso e con ragione fargli, èssi trovato, come ho detto, strumenti coi quali a volergli poi fare non si puo errare, pero che v'è la ragione.
>
> Come che chi volesse fare uno corpo quadro è mestiere avere uno di questi strumenti, o vuoi dire misura, e questa si quella che si chiama squadra, sanza la quale non giusto si puo fare uno corpo quadro....
>
> A volerlo fare sperico, non giusto si puo fare senza sesto, cioè tondo. E questo e l'altro strumento sanza il quale non si potrebbe fare.

[25] Ibid., p. 643:

> A voler fare ora questo quadro e questo tondo per via di scorcio, cioè per dimostrazione di disegno, dove che le parti, bench'elle sieno equali, ma all'occhio, perché non le puo giudicare tutte, non possono parere, e niente di meno sono, si che per volere fare questo è mesttiere pigliare questo ordine di questo solo punto il quale stimerai sia l'occhio e razzo visivo, de' quali si tratterà in processo.

[26] Ibid., p. 652:

> E benché l'uno ti paia largo e l'altro stretto , non curare....

E tutto questo piano ti verrà pieno di parelli, cioè quadretti d'uno braccio l'uno; e benché paino minore l'uno che l'altro, e anche non paino quadri, niente di meno sono tutti equali e quadri d'una medesima ragione, come che in processo si vedrà.

[27] Piero della Francesca, *De prospectiva pingendi*, ed. G. Nicco Fasola, Florence:Sansoni, 1942, p.64:

> La quinta è uno termine nel quale l'ochio descrive co'suoi raggi le cose proportionalmente et posse in quello giudicare la loro mesura: se non ci fusse termine non se poria intendere qunato le cose degrassaro, si che non se poieno dimostrare.

[28] Ibid., p.129:

> Dico che la prospectiva sona nel nome suo commo dire cose vedute da lungi, rapresentate socto certi dati termini con proportione, secondo la quantita de le distantie loro, senza de la quale non se po alcuna cosa degradare giustamente. Et perchè la pictura non è se non dimostrationi de superficie e de corpi degradati o acresciuti nel termine, posti secondo che le cose vere vedute da l'occhio socto diversi angolis'apresentano nel dicto termine....e non posendo giudicare da se lo intellecto la loro mesura, cioè quanto sia la piu propinqua et quanto sia la piu remota, pero dico essere necessaria la prospectiva, la quale discerne tucte le quantità proportionalmente commo vera scientia, dimostrando il degradare et acrescere de onni quantità per forza di linee.

[29] Leonardo da Vinci, *I manoscritti di Leonardo da Vinci : Il codice A (2172) nell'istituto di Francia*, Rome: Libreria dello Stato, 1936-1938, fol. 1^{v}:

> Pariete di vetro
>
> Prospettiva none altro che vedere uno sito djrieto a uno vetro piano epen transsparente sula superfitie del quale sia segniato tutte le chose che sono daesso vetro indirieto le quali si posano chondure perpiramide alpunto dellochio e essi piramide si tagliano sudetto vetro.
>
> Ibid., fol 3^{v}: “Prospettiva e ragione djmostrativa perla quale la sperientia conferma tutte le chose mandare allochio per linje piramjdali la lor simjlitudjne.”

[30] Cf. Ibid. *Les manuscrits de Léonard de Vinci; les manuscrits C, E, K*, ed. C. Ravaisson-Mollien, Paris: Quantin, 1888, fol 27^{v}:

> Prospettiva agiugnie dove mancha ilgiuditio nelle chose che djminnuiscano.....
>
> Lochio non potra maj essere vero judjce a terminare con verita quanto una quantita sotto vicina aunaltra simile laquale altra sia chola sua sommita alpari dellochio rigiardatore desse parti se non per mezo della pariete maestra e guida della prospettiva.

For a more complete analysis see the author's *Leonardo da Vinci Studies I: Linear perspective and the visual dimensions of science and art*, Munich: Deutscher Kunstverlag, 1986, pp. 68-86 and pp. 321-325.

[31] Luca Pacioli, *Summa di arithmetica, geometria, proportioni e proportionalita*, Venice: Paganinus de Paganinis, 1494, Preface:

> La perspectiva se ben si guarda senca dubio nulla sarebbe se queste non li se accomodasse. Cioe apieno dimostra el monarcha ali tempi nostri de la pictura

> maestro Pietro di franceschi nostro conterraneo... Cioe qui in vinegia Gentil e Giovan bellini carnal fratelli. E in perspectivo desegno Hyeronimo Malatini. E in Fiorenza Alexandro Boticelli, Philippino e Domenico grilandaio. E in peroscia Pietro ditto el perusino. E in Cortona Luca del nostro Maestro Piero degno discipul. E in Mantua Andrea Mantegna. E in Furli Melocco con suo caro alievo, Marco Palmezzano. Quali sempre con libella e circino lor opera proportionando a perfection mirabile conducano. In modo che non humane ma divine negli ochi nostri sapresentano....

[32] Vitruvius, *Architettura con il suo commento,* ed. M. Gianbatista Caporali, Perugia: Iano Biganzzini, 1536, fol. 6r:

> Hora sia il sesto stato trovato da chi el sia che piu necessario e stato alli mesuramenti di geometria, & prospettiva che a qualunque altro istrumento sia: perche con essu si mesurano tutte be liniali dimostrationi e le angularie cose alle quale si expetta le terminationi delle linie e divisioni...e tanto piu quanto e maggiore la multiplicatione per la inequalita de punti come e notissimo alli experti liniatori, di che specialmente sono di prospettiva.

[33] (Regiomontanus) Johannes Müller, *Scripta, clarissimi mathematici M. Ioannis Regiomontani, De Torqueto, Astrolabio armillari...aucta necessariis,* Nürnberg: Ioannem Montanum et Ulricum Neuber, 1544, p. aiiiv (in Regiomontanus, *Opera collectanea* , Osnabrück: Otto Zeller, 1972, p. 572):

> Porro nihil utilius tradi ppotest in tota doctrina de motibus corporum coelestium, his quinque instrumentis Astronomicis Torqueto, Astrolabio armillari, Regula magna Ptolemaica, Baculoque Astronomico et Quadrato Geometrico, uorum compositionem, et usum isti libelli continent, cum motibus Solis ac stellarum tam fixarum quam erraticarum, denique etiam Cometarum, iam olim per haec organam observatis.

[34] Regiomontanus, *Hec opera fient in oppido Nurimberga Germanie ductu Joannis de Monteregio*, c.1475. (in Regiomontanus, *Opera collectanea* , Osnabrück: Otto Zeller, 1972, p. 533):

> Radii visorii multurum generum cum usibus suis.
>
> De ponderibus et aquaeductibus cum figurationibus instrumentroum ad eas res necessariorum.
>
> De speculis ustoriis atque aliis multorum generum ususque stupendi.
>
> In officina fabrili astrarium in continuo tractatu est. Opus plane pro miraculo spectandum. Fiunt et alia instrumenta astronomica ad observationes caelestium itemque alia ad usum vulgarem quotidianum quorum nomina longum est recitare.

[35] For a survey of these problems and a brief history of the proportional compass or sector see: Ivo Schneider, *Der Proportionalzirkel. Ein universelles Analogrecheninstrument der Vergangenheit,* Munich: R. Oldenbourg Verlag 1970, (Deutsches Museum. Abhandlungen und Berichte, 38 Jg., 1970, Heft 2). On the problem of universal measurement see the author's "L'essor de l'esprit quantitative", *L'époque de la renaissance III (1520-1560),* ed. Eva Kushner, Budapest: Akademiai Kiado, (in press), and "Mesure quantification et science": *L'époque de la renaissance IV (1560-1600),* ed.

Tibor Klaniczay, Budapest: Akademiai Kiado, (in press). Cf. also: Paul Lawrence Rose, "The Origins of the proportional compass from Mordente to Galileo," *Physis*, Florence, anno X, fasc. 1, 1968, pp. 53-69; Fabrizio Mordente, *Il compasso del Signor Fabritio Mordente con altri istromenti mathematici ritrovati da Gasparo suo fratello*, Antwerp: apresso Cristofano Plantino, 1584 (Cf. manuscript copy, Munich, Bayerische Staatsbibliothek, Cod. it. 11). The development of these universal measuring instruments is the subject of a larger project by the author on *The Mastery of Quantity*.

[36] Jacques Bassentin, *Amplification de l'usage de l'astrolabe*, ed. J. Focard, Paris, 1555, p.51:

> Et pource qu'il n'est pas du tout possiblle que le sens et la raison puissent bien connoitre la vraye quantité de l'anglet aigu et variable, par ainsi il seroit tres difficile de naturellement comprendre la certaine quantité d'une chose, par la science de la perspective seulement. A ceste cause les anciens Geometriciens et mesureurs, ont inventé certains instrumens servans et faits pour cest art comme sont un cadran, un triangle geometrique, horloge manuel, quilindre et aultres desquelz l'usage serait long a declarer.

[37] See John Hammond, *The camera obscura. A chronicle*, Bristol: Adam Hilger Ltd., 1981 for a general treatment. For a concise article with citations from sources see J. Waterhouse, "Notes on the early history of the camera obscura," *Photographic journal*, London, vol. 25, 1901, pp. 270-290 and cf.: G. Pauschmann, "Zur Geschichte der linsenlosen Abbildung," *Archiv für Geschichte der der Mathematik, Naturwissenschaften und der Technik*, Leipzig, Bd. 9, 1922, 86-103. For mediaeval aspects of the theme see: "Guillaume de St. Cloud, Astronome," *Histoire littéraire de la France*, Paris, tom. XXV, 1869, p. 73; Maximilian Curtze, "Die Dunkelkammer, Eine Untersuchung uber die Vorgeschichte derselben," *Himmel und Erde*, Berlin, Jg. XIII, 1901,pp. 226- 232. See also that author's: "Die Abhandlung des Levi ben Gerson über Trigonometrie und den Jacobstab", *Bibliotheca Mathematica*, Stockholm, N. F., Bd. 12, no. 4, 1898, pp. 97-112. For Leonardo's activities in this context see the author's "Leonardo and the camera obscura" in: *Studi Vinciani in memoria di Nando de Toni*. Brescia: Ateneo di scienze lettere ed arti. Centro ricerche Leonardiane, 1986, pp. 81-92. For another of the early Renaissance decriptions see: Vitruvius, *De architectura*, ed. Cesare Cesariano, Como: 1521, fol. xxiii:

> Excellentemente tange una pulcherima ratione de optica quale fu experta et verificata dal Monastico Architecto Don Papnutio de Sancto Benedicto: si concavo al torno farai un circolo in qualche assicula di quantitate di uncie quatro vel sei, il concavo uncie due vel circa: et questa habia nel centro del concavo uno parvo et brevissimo spectaculo seu foramine quod scopus etiam dicitur: et infixo concordantemente in una valve seu anta di qualche fenestre clause per tal modo in lo loco dove sei non possa introire altra luce: et habi uno pocho di biancho papero vel altra cosa che recipia suso quello che si representera du epso in sino in tuta la terra et coelo sono contenuto.

[38] See the author's *Leonardo's Method*, Brescia: Ateneo di Brescia, (in press). These connections have been studied in detail in the author's as yet unpublished *Leonardo da Vinci Studies II*.